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Viscous Theory of Surface Noise Interaction Phenomena

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Viscous Theory of Surface Noise Interaction Phenomena

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SUMMARY

A generalized linear surface noise interaction problem is formulated. Noise production by an oscillating surface, turbulent or vortical interaction with a surface and scattering of sound by a surface are included in the generalized treatment. The direct effect of viscosity is included in the perturbation equations and the boundary conditions at the surface. An energy equation is derived to illustrate how the work or virtual work done at the interface is partitioned into the acoustic and vortical modes.

The problem is re-expressed in terms of a near field incompressible viscous problem for the Bernoulli enthalpy and a problem for the compressible but inviscid acoustic potential. A viscous integral equation for the Bernoulli enthalpy (or surface load) is derived. It is shown that viscosity will lead to a "unique" solution of the integral equation without the Kutta condition or other auxiliary singularity requirement. The acoustic enthalpy is calculated by quadrature over the near field solution.

The results of inviscid two-dimensional airfoil theory are used to discuss the interactive noise problem in the limit of high reduced frequency and small Helmholtz number. The acoustic spectrum is directly proportional to the surface load. The extreme limits of a full Kutta condition and no Kutta condition (actually no vorticity production) are considered. It is shown that in the case of vortex interaction with the surface the noise produced with the full Kutta condition is 3 dB less than the no Kutta condition result. Also, the spectrum with Kutta condition decays monotonically with frequency while the corresponding result without the Kutta condition decays in an oscillatory manner. It is suggested that the difference in the high frequency spectra could be detected experimentally.

The results of a supplementary study of an oscillatory airfoil in a medium at rest are discussed. It is concluded that viscosity can be a controlling factor in analyses and experiments of surface noise interaction phenomena. It is further concluded that the effect of edge bluntness and viscosity must be included in the problem formulation to correctly calculate the interactive noise.

I. INTRODUCTION

In two previous reports (Refs. 1 and 2) the homentropic theory of aeroacoustics, based on a kinematic definition of sound and the concept of Bernoulli enthalpy, was formulated in detail and applied to a variety of problems that illustrate the three basic questions of "free flow" aeroacoustics (i.e., in the absence of solid boundaries). In the spirit of Chu and Kovasznay (Ref. 3) the modal concept has been stressed in all of the previous work. The problem of how to decouple the acoustic and vortical modes (to the extent that it is possible) of energy transport is one of the most interesting questions in the field of aeroacoustics.

For example, when the problem of noise production by a turbulent flow (e.g., a jet) is considered, it is customary to adopt the "Lighthill hypothesis". Because of differences in the basic definition of sound in the various aeroacoustic formulations and the level at which the Lighthill hypothesis is invoked, there have been serious arguments over what constitutes the "source" of sound. With the modal approach there is no "source" per se, and the application of the Lighthill hypothesis is tantamount to assuming a unilateral transfer of energy from the vortical mode to the acoustic mode. The only internal mechanism for acoustic feedback to the vortical mode is the coriolis coupling acceleration between the acoustic particle velocity and the vorticity of the primary flow (see Eq. (2.28) of Ref. 1). The coriolis mechanism is important in the problem of sound scattering by a steady vortical flow and the stimulated emission of sound from an unsteady vortical flow. Both of these problems were investigated in detail in Ref. 2. It was shown by explicit calculation that the broadband noise radiated from a discrete vortex array can be enhanced by an incident sound field. (For a concise summary of this work, see Ref. 4).

The distinction between acoustic and vortical energy transport and the coriolis coupling mechanism between the two modes is reasonably well understood in the description of the internal dynamics of a fluid medium. However, there remains the important problem of how these modes interact in the presence of solid boundaries, and in particular when the solid boundary has "sharp" edges. The important feature of the surface interaction problem is that viscosity becomes a controlling factor in the mode coupling at the surface of the body. It is the objective of this report to shed some light on the importance of viscosity in a class of aeroacoustic problems that involves thin airfoil like surfaces, with edges. Two experiments due to Brooks (Ref. 5) and Brooks and Hodgson (Ref. 6) will be used to guide and test the development of the theory.

NOMENCLATURE

a	free stream speed of sound
A_n	spectral coefficients of the surface load, see Eq. (2.81)
c	airfoil chord
c^*	compressibility, see Eqs. (2.72) and (2.73)
C	denotes chord of the surface S in the two- dimensional problem
C_{mn}	see Eq. (2.83)
E	energy density, see Eq. (2.9)
f	surface deflection along z -axis
\vec{f}	surface force distribution, see Eq. (2.47)
F	magnitude of the surface force, see Eq. (2.47)
\mathcal{E}_m	see Eq. (2.84)
G	Green's function, see Eqs. (2.75) and (2.76)
h	vertical distance of vortex from interactive surface
h'	perturbation enthalpy
\mathcal{H}	Bernoulli enthalpy
$\vec{i}, \vec{j}, \vec{k}$	unit vectors along x, y, z axes, see Fig. 1
$J_n(z)$	Bessel function
k	$\omega c / 2v_\infty$, reduced frequency
\tilde{k}	ω / a
$K_0(z)$	modified Bessel function
l	spanwise correlation length of the surface load
M	v_∞ / a , free stream Mach number

\vec{M}	momentum flux, see Eq. (2.10)
\vec{n}	unit normal to a surface
\tilde{q}	denotes Fourier transform of variable q with respect to time
Δq	$q(z = 0^-) - q(z = 0^+)$, jump in the dependent variable q across the plane $z = 0$, e.g., see Eq. (2.53)
Q	dissipation function, see Eq. (2.11)
R	see Eq. (2.77)
S	denotes a fixed or oscillating surface
S_f	outer boundary of the volume V
$S(\alpha)$	acoustic spectral function, see Eqs. (3.1) and (3.2)
t	time
$T_n(x)$	Chebyshev polynomial of the first kind
\vec{u}'	incompressible perturbation velocity
$U_n(x)$	Chebyshev polynomial of the second kind
\vec{v}'	perturbation velocity field
\vec{v}_d	disturbance velocity field
v_∞	free stream velocity
V	integration volume in energy integral
W	upwash function see Eq. (2.4)
x, y, z	Cartesian coordinates, see Fig. 1
\mathcal{I}	acoustic intensity, see Eqs. (3.36) and (3.37)
\mathcal{K}	kernel function, see Eqs. (2.65) and (2.66)
$\mathcal{K}_1(x)$	integrated kernel, see Eq. (2.85)
\mathcal{L}	total lift on two-dimensional surface, see Eqs. (3.16) and (3.10)
$\mathcal{L}(x)$	normalized surface load distribution
\mathcal{W}	surface work or virtual work, see Eq. (2.13)

\mathcal{W}_0	see Eqs. (3.28) and (3.34)
α	see Eq. (3.3)
β^2	$1 - M^2$
$\gamma(x)$	vortex strength distribution, see Eqs. (3.9) and (3.15)
Γ	circulation, see Eq. (3.16)
Γ_0	vortex strength, see Eq. (3.33)
$\delta(z)$	Dirac delta function
θ	far field directivity angle
κ	$\omega c/2a$, Helmholtz number
λ, λ^*	see Eq. (2.57)
Λ	see Eq. (2.79)
ν	kinematic viscosity
τ	see Eq. (3.35)
ϕ	acoustic potential
χ	hydrodynamic potential, see Eq. (3.7)
ω	frequency in Fourier transform, see Eq. (2.42)
$\vec{\omega}$	perturbation vorticity, see Eq. (2.21)
curl	vector curl operator
div	divergence
grad	vector gradient operator
∇^2	Laplace operator
∇_i^2	surface Laplace operator, see Eq. (2.60)
—	time average
	absolute value

II. GENERAL THEORY OF A CLASS OF LINEAR AEROACOUSTIC PROBLEMS

A. The Surface Interaction Problem

Below, a linear problem is formulated that will permit the investigation of three basic noise-surface interactions:

- 1) An oscillating surface;
- 2) An assumed vortical disturbance (e.g., turbulence) interacts with the surface (The Edge Noise Problem); and
- 3) An assumed acoustic disturbance interacts with the surface (The Diffraction Problem).

These problems are depicted schematically in Figure 1. The surface S may be finite or infinite and initially the surface is supposed to be of zero thickness. Later, in Section III, the effect of finite edge geometry, is discussed.

It is assumed that each of the three problems may be described with the theory of homentropic small disturbances from a uniform main stream. The appropriate perturbation equations are:

$$\frac{1}{a^2} \frac{Dh'}{Dt} + \text{div } \vec{v}' = 0 \quad (2.1)$$

$$\frac{D\vec{v}'}{Dt} + \text{grad } h' = v \nabla^2 \vec{v}' + \frac{v}{3} \text{grad } (\text{div } \vec{v}') \quad (2.2)$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_{\infty} \frac{\partial}{\partial x} \quad (2.3)$$

For the problem of an oscillating surface the boundary conditions on S are as follows:

$$\left. \begin{aligned} \vec{k} \cdot \vec{v}' &= w = \frac{Df}{Dt} \\ \vec{k} \times \vec{v}' &= 0 \end{aligned} \right\} \text{ on } z = 0^{\pm} \text{ in } S \quad (2.4)$$

where f is the transverse (along the z -axis) deflection of the surface. The boundary conditions for the other two problems can also be expressed in the form of Eq. (2.4). Denote the incident velocity field due to a vortical or acoustic disturbance in the absence of S by \vec{v}_d . Then the interactive disturbance field must be such that it cancels the velocity \vec{v}_d on S ; i.e.,

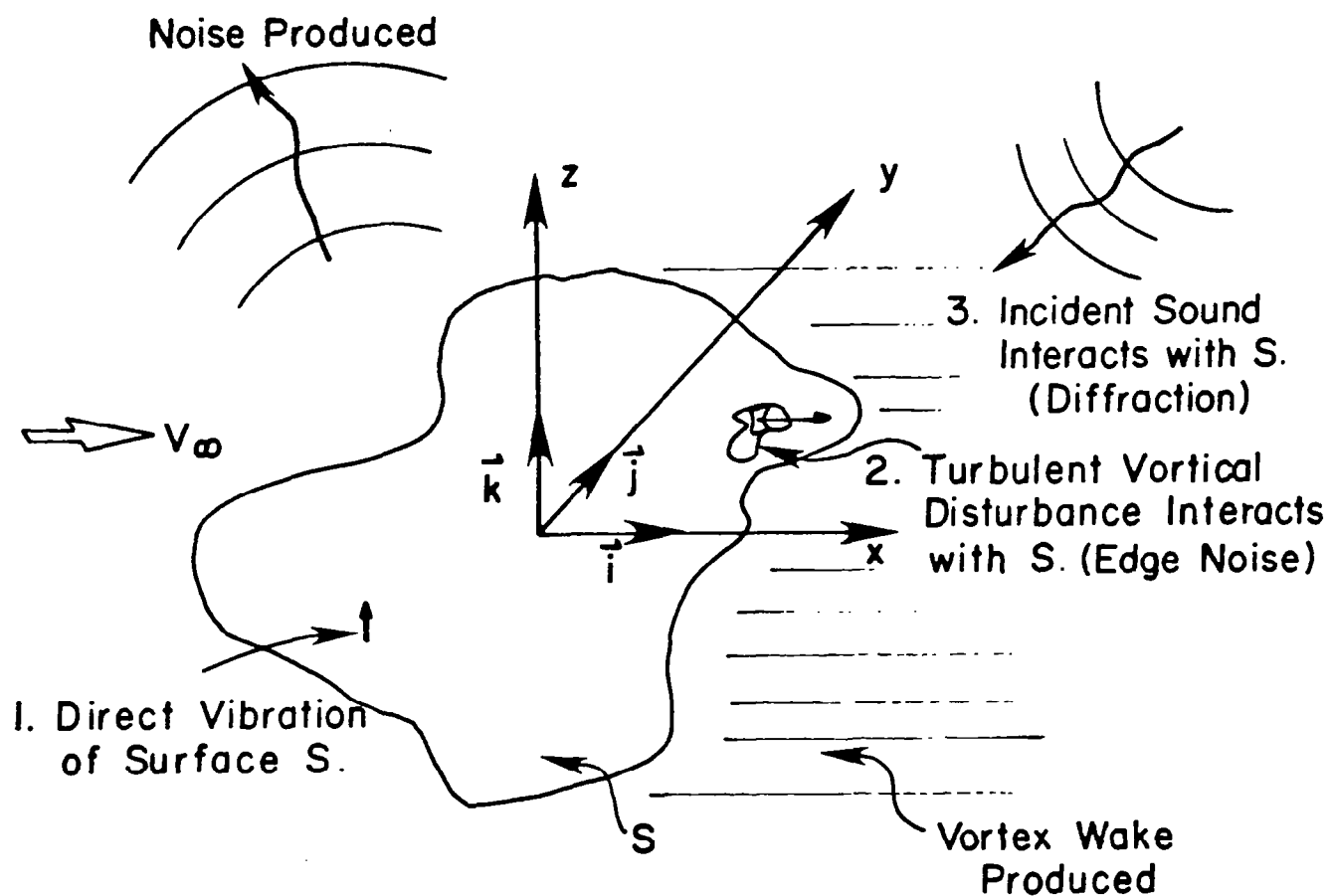


Figure 1 - Three Types of Interactive Noise Production Mechanisms

$$\left. \begin{aligned} \vec{k} \cdot \vec{v}' &= W = -\vec{k} \cdot \vec{v}_d \\ \vec{k} \times \vec{v}' &= -\vec{k} \times \vec{v}_d \end{aligned} \right\} \text{on } z = 0^\pm \text{ in } S \quad (2.5)$$

The boundary condition on the vertical, or z-component of velocity, is the usual boundary condition that is familiar in inviscid aerodynamic problems. The second boundary condition on the tangential surface perturbation velocity is necessary when viscous terms are retained in the momentum equation. The importance of the viscous terms and boundary conditions in establishing uniqueness of solution (without a Kutta condition) and in the partitioning of disturbance energy into the acoustic and vortical modes is the essential content of this report. Because of the linearity of the boundary value problem the solution can be obtained by superposition of two parts with the respective boundary conditions:

$$\left. \begin{aligned} \vec{k} \cdot \vec{v}' &= W = -\vec{k} \cdot \vec{v}_d \\ \vec{k} \times \vec{v}' &= 0 \end{aligned} \right\} \text{on } z = 0^\pm \text{ in } S \quad (2.6)$$

and

$$\left. \begin{aligned} \vec{k} \cdot \vec{v}' &= 0 \\ \vec{k} \times \vec{v}' &= -\vec{k} \times \vec{v}_d \end{aligned} \right\} \text{on } z = 0^\pm \text{ in } S \quad (2.7)$$

For all finite disturbance velocity fields \vec{v}_d the solution of Eqs. (2.1) and (2.2) with the boundary conditions Eq. (2.7) is small of the order of the viscosity and will not be considered further in this report. The homogeneous viscous boundary condition in Eq. (2.4) or (2.6) is the main result that is needed to formulate The generalized linear aero-acoustic problem. Solve the perturbation equations (2.1) and (2.2) subject to the boundary conditions (2.4) or (2.6) and the condition of outgoing decaying disturbances at infinity.

B. A Self Consistent Energy Equation

It is instructive to derive an energy equation for second order perturbation quantities. In so doing the global partitioning of energy into acoustic and vortical parts will become evident. Multiply Eq. (2.1) by h' , form the vector dot product of Eq. (2.2) with \vec{v}' and add the results to obtain the local energy balance,

$$\frac{DE}{Dt} + \text{div} \vec{M} = -Q \quad (2.8)$$

where

$$E = \frac{h'^2}{2a^2} + \frac{|\vec{v}'|^2}{2} \quad (2.9)$$

$$\vec{M} = h'\vec{v}' - v\vec{v}' \times \vec{\omega}' - \frac{4}{3}v\vec{v}' \operatorname{div} \vec{v}' \quad (2.10)$$

$$Q = v|\vec{\omega}'|^2 + \frac{4}{3}v(\operatorname{div} \vec{v}')^2 \quad (2.11)$$

For a statistically stationary disturbance velocity field, \vec{v}_d , the time average (denoted by an overbar) of Eq. (2.8) is

$$v_\infty \frac{\partial \bar{E}}{\partial x} + \operatorname{div} \bar{\vec{M}} = -\bar{Q} \quad (2.12)$$

Now integrate the last result over a large spherical volume, V , that encloses the surface S . On the outer boundary, S_f , the effect of viscous transport is assumed to be negligible. The resultant global energy balance becomes

$$\begin{aligned} \mathcal{W} &= 2 \int_S^+ \bar{M}_z dx dy \\ &= \int_{S_f} \bar{h'v'_n} dA + \int_{S_f} v_\infty \vec{i} \cdot \vec{n} \bar{E} dA \\ &\quad + \int_V \bar{Q} dV \end{aligned} \quad (2.13)$$

For the case of an oscillating airfoil, the first example of the generalized problem, the quantity \mathcal{W} is the rate at which work is done on the fluid by the oscillating surface. For the second and third examples the surface is rigid and does no work on the fluid. However, it is convenient to interpret \mathcal{W} as the virtual work due to the surface motion (upwash) W that must be imposed to counter the normal component of the disturbance velocity field. The global energy equation (2.13) shows that the work done by the surface S is partitioned into sound radiation through the outer surface S_f , convective energy transport through S_f , and viscous dissipation throughout the volume V . The last two parts are actually different manifestations of the same physical process; i.e., the formation of vorticity at the surface S . If the surface S_f is sufficiently far from S , no convective transport will be detected and all of the vortical energy will be counted in the

viscous dissipation. If S_f is closer to S then convective transport of vortical energy will be a significant part of the energy balance. It is noted that similar statements can be made about the acoustic term in Eq. (2.13). Certainly if S_f is removed to infinity then all of the surface work is dissipated by viscosity.

Finally it is pointed out that viscous dissipation cannot be discarded as an energy sink by the simple argument that the viscosity is small. It will be shown by explicit calculation in Section III and in the supplementary report that the vorticity is sufficiently singular to yield a finite integrated value for the dissipation even for vanishing small viscosity.

C. Mode Splitting and Boundary Conditions

The boundary value problem composed of equations (2.1), (2.2) and the generalized boundary conditions (2.4) or (2.6) is well posed. It is of interest, however, to recast the problem in terms of acoustic and vortical modes as follows:

Let

$$\vec{v}' = \text{grad } \phi + \vec{u}' \quad (2.14)$$

where ϕ is the acoustic potential and \vec{u}' is an incompressible velocity field. Substitute Eq. (2.14) into (2.2) to obtain

$$\frac{D\vec{u}'}{Dt} + \text{grad } \mathcal{H} = \nu \nabla^2 \vec{u}' \quad (2.15)$$

where

$$\mathcal{H} = h' + \frac{D\phi}{Dt} - \frac{4}{3} \frac{\nu}{a^2} \nabla^2 \phi \quad (2.16)$$

is the Bernoulli enthalpy (or Pseudo Sound) associated with the incompressible problem for \vec{u}' . Now use Eq. (2.16) to eliminate h' in Eq. (2.1). The result is a wave equation for ϕ ; i.e.,

$$\frac{1}{a^2} \frac{D^2 \phi}{Dt^2} - \frac{4}{3} \frac{\nu}{a^2} \nabla^2 \frac{D\phi}{Dt} - \nabla^2 \phi = \frac{1}{a^2} \frac{D\mathcal{H}}{Dt} \quad (2.17)$$

Note that, equations (2.15) and (2.17) could also be obtained by direct linearization of Eqs. (2.27) and (2.28) in Ref. 1. The acoustic mode is "driven" in the near field by the substantive rate of change of \mathcal{H} . In the far field the

acoustic mode decays due to viscous dilatational damping. The near field hydrodynamic or vortical mode is not coupled linearly to the acoustic mode in the interior of the gas. However, mode coupling does occur due to the boundary conditions at the surface. (See Ref. 7 for further discussion of this point).

The boundary conditions on \vec{u}', H are chosen to be the same as those for the complete perturbation velocity field; i.e.,

$$\left. \begin{aligned} \vec{k} \cdot \vec{u}' &= w \\ \vec{k} \times \vec{u}' &= 0 \end{aligned} \right\} \text{ on } z = 0^{\pm} \text{ in } S \quad (2.18)$$

Then the boundary conditions for ϕ are homogeneous; i.e.,

$$\left. \begin{aligned} \vec{k} \cdot \text{grad}\phi &= 0 \\ \vec{k} \times \text{grad}\phi &= 0 \end{aligned} \right\} \text{ on } z = 0^{\pm} \text{ in } S \quad (2.19)$$

It is possible to apply the no-slip boundary conditions on \vec{u}' because of the viscous term in Eq. (2.15). Similarly it is possible to apply no-slip conditions on the acoustic particle velocity as long as the viscous damping term is retained in Eq. (2.17). It is the presence of these terms that precludes the necessity for a Kutta condition or other singularity criteria to solve the relevant boundary value problem.

A somewhat different insight into the role of viscosity and the coupling of the pressure and vorticity modes may be obtained as follows. Take the curl of Eq. (2.2) or (2.15) to obtain the vorticity diffusion equation

$$\frac{D\vec{\omega}'}{Dt} = \nu \nabla^2 \vec{\omega}' \quad (2.20)$$

with

$$\vec{\omega}' = \text{curl} \vec{v}' = \text{curl} \vec{u}' \quad (2.21)$$

Also take the substantive derivative of Eq. (2.1) and the divergence of (2.2) to obtain the damped wave equation for h' ; i.e.,

$$\frac{1}{a^2} \frac{D^2 h'}{Dt^2} - \frac{4}{3} \frac{\nu}{a^2} \nabla^2 \frac{Dh'}{Dt} - \nabla^2 h' = 0 \quad (2.22)$$

As noted by Chu and Kovasznay (Ref. 3) the vorticity and pressure modes decouple completely in the interior of the gas. At the surface, the boundary conditions on \vec{v}' can be used to derive the relations between h' and $\vec{\omega}'$. First write Eq. (2.2) in the form,

$$\frac{D\vec{v}'}{Dt} + \text{grad} \left(h' + \frac{4}{3} \frac{\nu}{a^2} \frac{Dh'}{Dt} \right) = - \nu \text{curl } \vec{\omega}' \quad (2.23)$$

Because of the planar geometry of S and the no-slip boundary conditions, it follows that

$$\omega_z = \vec{k} \cdot \vec{\omega}' = 0 \quad (2.24)$$

Now project Eq. (2.23) onto the surface S and use the boundary conditions on \vec{v}' to obtain the following boundary conditions.

$$\begin{aligned} \frac{\partial}{\partial z} \left(h' + \frac{4}{3} \frac{\nu}{a^2} \frac{Dh'}{Dt} \right) &= - \frac{Dw}{Dt} + \nu \left(\frac{\partial \omega_x}{\partial y} - \frac{\partial \omega_y}{\partial x} \right) \\ \vec{k} \times \text{grad} \left(h' + \frac{4}{3} \frac{\nu}{a^2} \frac{Dh'}{Dt} \right) &= \nu \frac{\partial \vec{\omega}'}{\partial z} \quad \text{on } z = 0^\pm \text{ in } S \end{aligned} \quad (2.25)$$

The intrinsic coupling of the pressure and vorticity modes is evident from these relations. The normal pressure gradient at the surface is balanced by the surface acceleration (real or virtual) and gradients of the surface vorticity. The second expression shows how vorticity is produced at a surface by pressure gradients along the surface. Near sharp edges that have large pressure gradients, the production of vorticity can be intense. It is also clear from this relation that to obtain a meaningful solution (non-constant surface load) of the viscous boundary value problem, the product $\nu(\partial \vec{\omega}' / \partial z)$ must be finite even for a vanishingly small viscosity coefficient. The coupling between modes is thus very singular.

D. The Associated Inviscid Boundary Value Problem

For each of the boundary value problems and energy relations derived thus far there is a corresponding "inviscid" relation. First of all, the perturbation enthalpy and velocity satisfy the relations

$$\frac{1}{a^2} \frac{Dh'}{Dt} + \text{div } \vec{v}' = 0 \quad (2.26)$$

$$\frac{D\vec{v}'}{Dt} + \text{grad } h' = 0 \quad (2.27)$$

with the boundary condition

$$\vec{k} \cdot \vec{v}' = 0 \quad \text{on } z = 0^+ \text{ in } S \quad (2.28)$$

and the radiation condition. The energy relation (2.13) becomes

$$\begin{aligned} \mathcal{W} &= 2 \int_{S^+} \overline{h'W} \, dx dy \\ &= \int_{S_f} \overline{h'v_n'} \, dA + \int_{S_f} v_\infty \vec{i} \cdot \vec{n} \, E \, dA \end{aligned} \quad (2.29)$$

where the outer surface S_f can be taken to infinity since acoustic and vortical waves do not decay. The convective transport of energy in Eq. (2.29) is due to the singular vortex wake. It is the singular remnant of the effect of viscosity, and in general it is a significant part of the work done by the surface. Without the explicit effect of viscosity at the surface there is no way to determine the energy partitioning without recourse to an artificial uniqueness criteria like the Kutta condition. This will be demonstrated explicitly in the next section where the boundary value problem is cast in the form of a singular integral equation.

The inviscid boundary value problems for \vec{u}' and ϕ are respectively:

$$\text{div } \vec{u}' = 0 \quad (2.30)$$

$$\frac{D\vec{u}'}{Dt} + \text{grad } \mathcal{H} = 0 \quad (2.31)$$

$$\vec{k} \cdot \vec{u}' = W \text{ on } z = 0^\pm \text{ in } S \quad (2.32)$$

and

$$\frac{1}{a^2} \frac{D^2 \phi}{Dt^2} - \nabla^2 \phi = \frac{1}{a^2} \frac{D\mathcal{H}}{Dt} \quad (2.33)$$

with

$$\frac{\partial \phi}{\partial z} = 0 \text{ on } z = 0^\pm \text{ in } S \quad (2.34)$$

The perturbation enthalpy

$$h' = \mathcal{H} - \frac{D\phi}{Dt} \quad (2.35)$$

also satisfies the homogeneous wave equation

$$\frac{1}{a^2} \frac{D^2 h'}{Dt^2} - \nabla^2 h' = 0 \quad (2.36)$$

and

$$\frac{D\vec{\omega}'}{Dt} = 0 \quad (2.37)$$

It follows from (2.25) that

$$\frac{\partial h'}{\partial z} = - \frac{DW}{Dt} \text{ on } z = 0^\pm \text{ in } S \quad (2.38)$$

In each boundary value problem the boundary conditions on the tangential surface velocity components must be given up because the inviscid equations are of lower order. Also, the inviscid problems do not have unique solutions and some auxiliary criterion (usually the Kutta condition) must be used to establish uniqueness.

E. An Integral Equation for Bernoulli Enthalpy (Uniqueness)

Consider the viscous boundary value problem for \vec{u}' and \mathcal{H} ; i.e.,

$$\operatorname{div} \vec{u}' = 0 \quad (2.39)$$

$$\frac{D\vec{u}'}{Dt} + \operatorname{grad} \mathcal{H} = \nu \nabla^2 \vec{u}' \quad (2.40)$$

$$\left. \begin{array}{l} \vec{k} \cdot \vec{u}' = w \\ \vec{k} \times \vec{u}' = 0 \end{array} \right\} \quad \text{on } z = 0^\pm \text{ in } S \quad (2.41)$$

It is convenient to work in the frequency domain. Take Fourier transforms of the above equations with respect to time and note for any dependent variable, q , that

$$\underline{q}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} q(t) dt$$

$$q(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} \underline{q}(\omega) d\omega \quad (2.42)$$

Then

$$\operatorname{div} \vec{u}' = 0 \quad (2.43)$$

$$\frac{D\vec{u}'}{Dx} + \operatorname{grad} \mathcal{H} = \nu \nabla^2 \vec{u}' \quad (2.44)$$

with

$$\left. \begin{array}{l} \vec{k} \cdot \vec{u}' = \underline{w} \\ \vec{k} \times \vec{u}' = 0 \end{array} \right\} \quad \text{on } z = 0^\pm \text{ in } S \quad (2.45)$$

and

$$\frac{D}{Dx} = v_\infty \frac{\partial}{\partial x} + i\omega \quad (2.46)$$

The solution can be represented in terms of an unknown normal force distribution on the surface S ; i.e.,

$$\vec{f} = -\vec{k} \underline{F}(x,y) \delta(z) \quad (2.47)$$

Add \vec{f} to the right hand side of (2.44) and then take the divergence and curl to obtain

$$\nabla^2 \underline{H} = -\underline{F} \frac{\partial \delta}{\partial z} \quad (2.48)$$

and for the vorticity

$$\frac{D\vec{\omega}'}{Dt} - \nu \nabla^2 \vec{\omega}' = \vec{k} \times \text{grad} \underline{F} \delta(z) \quad (2.49)$$

The solution of (2.48) is

$$\underline{H} = \frac{1}{4\pi} \frac{\partial}{\partial z} \int_S \frac{\underline{F}(\vec{\xi}) d\vec{\xi}}{|\vec{x} - \vec{\xi}|} \quad (2.50)$$

where the integration is over the surface S with

$$\vec{\xi} = \vec{i}\xi + \vec{j}\eta, \quad d\vec{\xi} = d\xi d\eta \quad (2.51)$$

Take the limit of (2.50) as $z \rightarrow 0$ to obtain

$$\underline{H}^{\pm} = \lim_{z \rightarrow 0^{\pm}} \underline{H}_{\pm} = \mp \frac{\underline{F}}{2} \quad (2.52)$$

or

$$\underline{F} = \Delta \underline{H} = \underline{H}(x,y,0^-) - \underline{H}(x,y,0^+) \quad (2.53)$$

is the local jump in the Bernoulli enthalpy on the surface. For points off the surface $\Delta \underline{H}$ must vanish.

The solution of (2.49) can be expressed in the form

$$\vec{\omega}' = \vec{k} \times \text{grad} Q \quad (2.54)$$

where

$$\frac{DQ}{Dx} - v \nabla^2 Q = \underline{F} \delta(z) \quad (2.55)$$

and it is readily shown that

$$Q = \frac{1}{4\pi v} \int_S \underline{F}(\vec{\xi}) \frac{\exp[\lambda \vec{f} \cdot (\vec{x} - \vec{\xi}) - \lambda^* |\vec{x} - \vec{\xi}|]}{|\vec{x} - \vec{\xi}|} d\vec{\xi} \quad (2.56)$$

where

$$\lambda^* = (\lambda^2 + i\omega/v)^{\frac{1}{2}}, \quad \lambda = \frac{v_\infty}{2v} \quad (2.57)$$

and

$$\text{curl} \vec{\omega}' = \vec{k} \nabla^2 Q - \text{grad} \frac{\partial Q}{\partial z} \quad (2.58)$$

From (2.44), with $z \neq 0$

$$\begin{aligned} \frac{DW}{Dx} + \frac{\partial \mathcal{H}}{\partial z} &= -v \vec{k} \cdot \text{curl} \vec{\omega}' \\ &= -v \nabla_1^2 Q \end{aligned} \quad (2.59)$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (2.60)$$

Also

$$\frac{\partial \mathcal{H}}{\partial z} = \nabla_1^2 \frac{1}{4\pi} \int_S \frac{\underline{F}(\vec{\xi})}{|\vec{x} - \vec{\xi}|} d\vec{\xi} \quad (2.61)$$

Thus

$$\begin{aligned} \frac{DW}{Dx} &= \frac{1}{4\pi} \int_S \underline{F}(\vec{\xi}) \nabla_1^2 \frac{1 - \exp[\lambda \vec{f} \cdot (\vec{x} - \vec{\xi}) - \lambda^* |\vec{x} - \vec{\xi}|]}{|\vec{x} - \vec{\xi}|} d\vec{\xi} \\ &\quad \text{for } z \neq 0 \end{aligned} \quad (2.62)$$

Now solve (2.62) for W and take the limit as $z \rightarrow 0$ to obtain

the following integral equation for $\underline{f}(\vec{\xi})$; i.e.,

$$\int_S \underline{f}(\vec{\xi}) \mathcal{K}(\vec{x} - \vec{\xi}) d\vec{\xi} = \underline{W}(\vec{\xi}), \vec{x} \text{ in } S \quad (2.63)$$

where the kernel is defined by the relation

$$\mathcal{K}(\vec{x}) = \frac{1}{4\pi v_\infty} \nabla^2 \int_0^\infty \exp(-iws/v_\infty) \frac{1 - \exp\{\lambda(x-s) - \lambda^*|\vec{x} - \vec{r}_s|\}}{|\vec{x} - \vec{r}_s|} ds \quad (2.64)$$

The integral equation for the corresponding two-dimensional case can be obtained by integrating (2.64) over the spanwise coordinate. From a previous report (Ref. 8) the final expression is

$$\int_C \underline{f}(\xi) \mathcal{K}(x - \xi) d\xi = - \underline{W}(\xi), x \text{ in } C \quad (2.65)$$

where

$$\mathcal{K}(x) = \frac{1}{2\pi v_\infty} \frac{\partial^2}{\partial x^2} \int_0^\infty \exp(-iws/v_\infty) [\ln|x - \xi| + \exp\{\lambda(x - \xi)\} K_0(\lambda^*|x - \xi|)] d\xi \quad (2.66)$$

and the integration is only over the chord C of the interactive surface S .

The solution of (2.65) and a more general equation for airfoil type surfaces with finite thickness has been the subject of a recent development of "viscous thin airfoil theory". The results are summarized in a final report (Ref. 8) and an AGARD report (Ref. 9). For the moment the most important point to note is that the kernel function $\mathcal{K}(x)$ (Eq. (2.66)) has a logarithmic singularity for small argument no matter how small the coefficient of viscosity. The corresponding inviscid kernel of the incompressible Possio equation (obtained by dropping the second term in (2.66)) has a Cauchy singularity. The viscous integral equation has a unique solution (See Ref. 8) while the inviscid equation has an eigensolution that

is associated with the circulation of the perturbation flow field (See Section IIIB). The strength of the eigensolution determines uniquely the amount of vortical energy that is shed into the wake. The eigensolution also contributes directly to the "source" term DH/Dt in the acoustic problem (2.33) and (2.34) and so affects the radiated sound. Without viscosity there is no physical criterion to determine the eigensolution and it is customary to resort to the Kutta condition or principle of minimum singularity at the trailing edge to establish uniqueness. The solution of the viscous integral equation (2.66) has a square root singularity at a mathematically sharp leading or trailing edge. The singularities can be removed by slightly blunting the edges as shown in Section III and in that case there is no useful edge criteria whatsoever to establish uniqueness. The surface load, shed vorticity and noise must be obtained by solving the viscous boundary value problem.

F. The Acoustic Potential

Once H is known from the solution of the appropriate integral equation, there remains the problem of solving for the acoustic potential ϕ ; i.e.,

$$\frac{1}{a^2} \frac{D^2 \phi}{Dt^2} - \frac{4}{3} \frac{v}{a^2} \nabla^2 \frac{D\phi}{Dt} - \nabla^2 \phi = \frac{1}{a^2} \frac{DH}{Dt} \quad (2.67)$$

with

$$\left. \begin{aligned} \frac{\partial \phi}{\partial z} &= 0 \\ \vec{k} \times \text{grad} \phi &= 0 \end{aligned} \right\} \text{ on } z = 0^+ \text{ in } S \quad (2.68)$$

and the radiation condition. Actually, because of the dilatational damping term, it is sufficient to require decaying solutions of (2.67) at infinity.

The interpretation of (2.67) and (2.68) is conceptually appealing. The origin of the interactive noise is the local substantive rate of change of H . The surface itself is a passive element in the noise production process but it does cause compressible diffraction primarily through the boundary condition on the induced vertical component of velocity. Also, it is emphasized that by construction no vorticity is produced in the compressible part of the interaction process. All vorticity is associated with the surface loading due to H . It is this observation that permits an important simplification of the acoustic problem; i.e., the direct effect of

viscosity can be omitted in the acoustic problem. Since there must be no compressible production of vorticity, the compressible viscous term is omitted in (2.67) and the viscous boundary condition in (2.68) is replaced by the condition that the acoustic potential be continuous across the plane $z = 0$ for all points not in S ; i.e.,

$$\frac{1}{a^2} \frac{D^2 \phi}{Dt^2} - \nabla^2 \phi = \frac{1}{a^2} \frac{DH}{Dt} \quad (2.69)$$

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{on } z = 0^\pm \quad \text{for } (x, y) \text{ in } S \quad (2.70)$$

$$\phi(z = 0^+) = \phi(z = 0^-) \quad \text{for } (x, y) \text{ not in } S \quad (2.71)$$

plus a radiation condition on the far field behavior of ϕ .

Now take the Laplacian of (2.69) and introduce the compressibility variable c^* such that

$$\nabla^2 \phi = c^* \quad (2.72)$$

Then, in the frequency domain

$$\begin{aligned} \frac{1}{a^2} \frac{D^2 \zeta^*}{Dx^2} - \nabla^2 \zeta^* &= \frac{1}{a^2} \frac{D}{Dx} \nabla^2 \mathcal{H} \\ &= - \frac{1}{a^2} \frac{DF}{Dx} \frac{\partial \delta}{\partial z} \end{aligned} \quad (2.73)$$

where (2.48) has been used to replace $\nabla^2 \mathcal{H}$. The solution for ζ^* is

$$\zeta^* = \frac{1}{a^2} \frac{D}{Dx} \frac{\partial}{\partial z} \int_S \mathcal{F}(\vec{\xi}) G(x - \vec{\xi}) d\vec{\xi} \quad (2.74)$$

where

$$\frac{1}{a^2} \frac{D^2 G}{Dx^2} - \nabla^2 G = \delta(\vec{x}) \quad (2.75)$$

or

$$G = \frac{\exp(-i\tilde{k}/\beta^2)(R - Mx)}{4\pi R} \quad (2.76)$$

with

$$\begin{aligned}
 R &= \left[x^2 + \beta^2 (y^2 + z^2) \right]^{\frac{1}{2}} \\
 \beta^2 &= 1 - M^2 \\
 M &= v_{\infty}/a \\
 \tilde{k} &= \omega/a
 \end{aligned} \tag{2.77}$$

In the far field

$$\varphi^* \cong \frac{1}{a^2} \frac{D}{Dx} \frac{\partial}{\partial z} G(\vec{x}) \int_S \exp(i\tilde{k}\Lambda/\beta^2) \tilde{F}(\vec{\xi}) d\vec{\xi} \tag{2.78}$$

with

$$\Lambda = \left(\frac{x}{R} - M \right) \xi + \frac{\beta^2 y}{R} \eta \tag{2.79}$$

From the continuity equation (2.1) it follows that the far field acoustic enthalpy is the compressible continuation of H via the relation

$$h_a' = - \frac{\partial G(\vec{x})}{\partial z} \int_S \exp(i\tilde{k}\Lambda/\beta^2) \tilde{F}(\vec{\xi}) d\vec{\xi} \tag{2.80}$$

Thus it appears that the far field pressure and the OASPL for example can be calculated from the incompressible problem without specifically solving for the acoustic potential. The actual acoustic potential and velocity and therefore the far field intensity must include the solution of (2.72) subject to the diffraction boundary conditions (2.70) and (2.71).

The essential effect of viscosity, then, is to establish "uniqueness" of solution of the surface load distribution in the integral equation (2.65) with the kernel function (2.66). Once the surface loading is determined, the shed vorticity and the acoustic far field are uniquely determined. The same statement can be made about the corresponding inviscid problem (See Section IID). However, the corresponding compressible or incompressible integral equation for the surface load has an eigensolution. The strength of the acoustic field and the vortex wake depend on the strength of the

eigensolution. Without a viscous integral equation to determine the surface reactive load there is no apriori physical criterion to determine the eigensolution. It is then customary to invoke the Kutta condition, principle of minimum singularity or modifications thereof to establish uniqueness. It is important to note that edge singularities are completely admissible in the linearized viscous or inviscid problem. Only by accounting for finite edge curvature can these singularities be eliminated. It is also true that the edge singularities pose no real problem in the calculation of the noise or vortex wake. It is much more important to calculate the surface loading correctly, both in magnitude and phase, than to worry about the appearance or absence of an edge singularity. These remarks, by the way, are independent of frequency. Even for very low frequency interactions where the noise and wake vorticity scale with the total lift, it is known (Ref. 8) that the Kutta condition will usually overestimate the lift.

G. Chebyshev Representation of the Surface Load and Acoustic Far Field

Consider the class of quasi two-dimensional problems where the surface load is calculated from (2.70) and the far field from (2.79) with the effective spanwise length scale l over which the load is correlated or non-zero. The solution of the integral equation (2.65) can be expressed in a series of Chebyshev polynomials; i.e.,

$$\tilde{F}(x) = \sum_{n=0}^{\infty} A_n \frac{T_n(2x/c)}{[1 - (2x/c)^2]^{\frac{1}{2}}} \quad (2.81)$$

where c is the chord of the two-dimensional surface. The spectral coefficients A_n may be calculated from the finite matrix representation of the integral equation; i.e.,

$$\sum_{n=0}^N C_{mn} A_n = g_m \quad m=0,1,\dots,N \quad (2.82)$$

with

$$C_{mn} = \frac{(m+1)}{\pi^2} \int_0^\pi \cos n\phi d\phi \int_0^\pi \cos\{(m+1)\theta\} d\theta \mathcal{K}_1(\cos\theta - \cos\phi) \quad (2.83)$$

$$g_m = -\frac{2}{\pi} \int_{-1}^1 (1-x^2)^{1/2} U_m(x) W(x) dx \quad (2.84)$$

and

$$\mathcal{K}_1(x) = 2\pi v_\infty \int_{-\infty}^x \mathcal{K}(\xi) d\xi \quad (2.85)$$

The numerical solution of (2.82) for the surface load distribution is the subject of Ref. 8.

Now substitute the Chebyshev series (2.81) into (2.80) to obtain the acoustic far field in terms of the spectral coefficients of the surface load. The result is

$$\begin{aligned} h_a' &= -\frac{\partial G}{\partial z} \cdot \ell \cdot \int_{-c/2}^{c/2} \exp[(i\tilde{\kappa}/\beta^2)(x/R - M)] \xi \\ &\quad \sum_{n=0}^{\infty} A_n \frac{T_n(2\xi/c)}{(1 - (2\xi/c)^2)^{1/2}} d\xi \\ &= -\frac{\partial G}{\partial z} \cdot \frac{\ell c}{2} \sum_{n=0}^{\infty} A_n \int_0^\pi \exp\{(i\kappa/\beta^2) \\ &\quad (x/R - M)\cos\theta\} \cos n\theta d\theta \\ &= \frac{i\kappa \cdot \ell}{8} (z/R) \exp\{(-i\kappa/\beta^2)(R - Mx)\} \\ &\quad \sum_{n=0}^{\infty} A_n i^n J_n\{(\kappa/\beta^2)(x/R - M)\} \end{aligned} \quad (2.86)$$

with the Helmholtz number

$$\kappa = \omega c/2a = \tilde{\kappa}c/2 \quad (2.87)$$

For the range of compact frequencies ($\kappa \ll 1$) only the

first term in the series of (2.86) contributes to the far field and the magnitude of h_a' is proportional to A_0 or the total interactive lift that develops on the surface. The value of A_0 may be calculated uniquely with the viscous theory as shown in Refs. 8 and 9.

H. Effect of Edge Bluntness

From extensive calculations of the solution of (2.82) in Ref. 9 the following important result was found. For a surface with zero thickness the edges are mathematically sharp. As a result, the solution of (2.82) converges to the inviscid solution with Kutta condition (See Section III) for any Reynolds number greater than about 500. The departure from inviscid behavior is of order $1/(Re)^{1/2}$. The reason is that by introducing a mathematically sharp edge or zero length scale the linear viscous effect is grossly overexaggerated at the edges. By introducing a finite edge radius the viscous edge effect is diminished and viscous action over the entire surface becomes the controlling factor in establishing the surface loading. In all cases calculated (See Ref. 8) the surface loading is diminished by introducing finite edge geometry. An example of the reduction in the steady state lift curve slope on a surface at angle of attack is shown in Figure 2. The lift on an elliptic section is about 15% less than the lift on a Joukowski airfoil with cusped trailing edge. More extensive comparisons of unsteady results for surfaces with and without edge curvature may be found in Ref. 8. For moderately low values of reduced frequency ($\omega c/2v_\infty =$ order of a few tenths) the real and imaginary parts of A_0 , A_1 , the two leading spectral coefficients in the surface load series (2.80) for an elliptic section are substantially different from those of flat plate theory. The changes are in quantitative agreement with measurements of Davis and Malcolm (Ref. 10).

To perform meaningful acoustic calculations it is necessary to solve the surface load problem for relatively large values of reduced frequency $\{ (\omega c/2v_\infty) = O(1/M) \text{ and larger } \}$. The numerical problems in evaluating the kernel functions with thickness and viscosity for high frequency have not been completely resolved and detailed results on this aspect of the problem will be reserved for a future study.

III. APPLICATIONS OF "INVISCID" THEORY

A. Zero Thickness Two-Dimensional Surface

In the following development the family of solutions of the well known two-dimensional incompressible oscillating airfoil

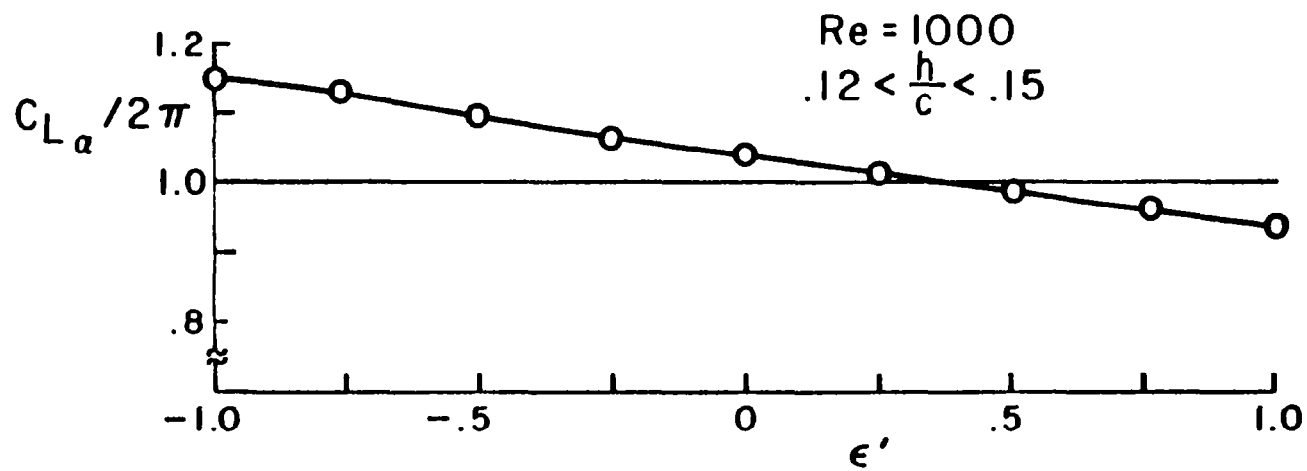
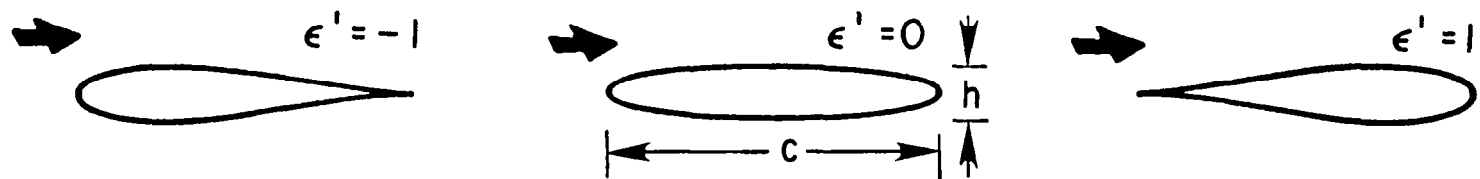


Figure 2 - Degradation of the Steady State Lift Curve Slope ($C_{L\alpha}$) due to Trailing Edge Bluntness (ϵ'); note the airfoils sketched above the graph.

problem (Ref. 11) is used to draw some general inferences about the generation of sound for the generalized surface interaction problems. Following the work of Yates and Houbolt (Ref. 12) and recent work of Howe (Ref. 13) the extreme limits of a full Kutta condition and no Kutta condition are used to illustrate why it is important to complete the development of the viscous theory as formulated in the previous Sections.

For the two-dimensional problem, the spectrum of the acoustic enthalpy (See Eq. (2.79)) can be expressed in the form

$$\underline{h}'_a \cong - \frac{\partial G}{\partial z} \frac{\ell c}{2} v_\infty^2 S(\alpha) \quad (3.1)$$

where G is the asymptotic form of the three-dimensional Green's function (2.76) and ℓ is some measure of the spanwise correlation of the surface disturbances. The airfoil semi-chord ($c/2$) is chosen as the unit of length and the spectral function $S(\alpha)$ is expressed in the form

$$S(\alpha) = \int_{-1}^1 \exp(i\alpha x) \mathcal{L}(x) dx \quad (3.2)$$

where

$$\alpha = (ik / \beta^2)(\cos\theta - M) \quad (3.3)$$

and $\mathcal{L}(x)$ is the normalized chordwise load distribution that must be calculated from the incompressible boundary value problem:

$$\text{div } \vec{u} = 0$$

$$\frac{D\vec{u}}{Dx} + \text{grad } P = 0$$

$$\vec{f} \cdot \vec{u} = \mathcal{W} \text{ on } y = 0, \quad -1 < x < 1 \quad (3.4)$$

with

$$\frac{D}{Dx} = \frac{\partial}{\partial x} + ik \quad (3.5)$$

and

$$\mathcal{L}(x) = P(x, 0^-) - P(x, 0^+) \quad (3.6)$$

The solution of (3.4) can be expressed in terms of a hydrodynamic potential χ and a vortex sheet distribution $\gamma(x)$ that are related to \mathcal{L} as follows:

$$\vec{u} = \text{grad } \chi \quad \text{for } y \neq 0 \quad (3.7)$$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \gamma(x) \delta(y) \quad (3.8)$$

$$\gamma(x) = \int_{-\infty}^{\infty} \omega \, dy = \frac{\partial \Delta x}{\partial x} \quad (3.9)$$

$$\mathcal{L} = - \frac{D \Delta x}{Dx} \quad (3.10)$$

with

$$\Delta x = \chi(x, 0^-) - \chi(x, 0^+) \quad (3.11)$$

The potential is a harmonic function with the representation

$$\chi = \frac{1}{2\pi} \int_{-1}^{\infty} \gamma(\xi) \tan^{-1} \frac{y}{(x - \xi)} \, d\xi \quad (3.12)$$

For $x > 1$ (in the wake) the following relations hold:

$$\mathcal{L}(x) = 0 \quad (3.13)$$

$$\Delta x = - \Gamma \exp\{-ik(x - 1)\} \quad (3.14)$$

$$\gamma(x) = - ik \Gamma \exp\{-ik(x - 1)\} \quad (3.15)$$

where Γ is the negative of the potential jump across the trailing edge. Also, Γ is the circulation around the surface; i.e.,

$$\Gamma = \oint_{\substack{y=0^- \\ y=0^+ \\ x=1}} \vec{u} \cdot d\vec{s} = - \Delta \chi \Big|_{x=1} \quad (3.16)$$

The value of Γ is unknown and in fact is completely indeterminate with inviscid theory.

An integral equation for $\gamma(x)$ is obtained by applying the boundary condition (3.4) to the integral representation (3.12). The result is

$$\frac{1}{\pi} \oint_{-1}^1 \frac{\gamma(\xi)}{(\xi - x)} d\xi = g(x) \quad (3.17)$$

with

$$g(x) = -2 \mathcal{W}(x) - i \frac{k\Gamma}{\pi} \int_1^\infty \frac{\exp\{-ik(\xi - 1)\}}{(\xi - x)} d\xi$$

where the slash through the integral denotes the Cauchy principle value. (3.18)

The solution of (3.17) is

$$\begin{aligned} \gamma(x) = & \frac{1}{\pi(1 - x^2)^{1/2}} \left[-\Gamma + \frac{ik\Gamma}{\pi} \oint_{-1}^1 \frac{(1 - s^2)^{1/2}}{s - x} ds \right. \\ & \left. + 2 \oint_{-1}^1 \frac{(1 - s^2)^{1/2}}{(s - x)} \mathcal{W}(s) ds \right] \end{aligned} \quad (3.19)$$

a well known result (e.g., See Ref. 11).

The spectral function $S(\alpha)$ from (3.2) can be expressed in terms of $\gamma(x)$ with the formula

$$S(\alpha) = \Gamma \frac{k}{\alpha} e^{i\alpha} + \left(\frac{k}{\alpha} - 1 \right) \int_{-1}^1 e^{i\alpha x} \gamma(x) dx \quad (3.20)$$

that is easily derived from (3.9) and (3.10) by integration by parts. Now substitute (3.19) into (3.20) to obtain the following representation of the acoustic spectrum:

$$\begin{aligned} S(\alpha) = & \Gamma \left[\left(1 - \frac{k}{\alpha} \right) J_0(\alpha) + \frac{k}{\alpha} e^{i\alpha} \right. \\ & + 2ik \left(1 - \frac{k}{\alpha} \right) \sum_{n=1}^{\infty} i^n J_n(\alpha) \int_1^{\infty} \frac{\exp\{-ik(\xi - 1)\}}{[\xi + (\xi^2 - 1)^{\frac{1}{2}}]^n} d\xi \Big] \\ & + 4 \left(1 - \frac{k}{\alpha} \right) \sum_{n=1}^{\infty} i^n J_n(\alpha) \int_{-1}^1 (1 - \xi^2)^{\frac{1}{2}} U_{n-1}(\xi) \mathcal{W}(\xi) d\xi \end{aligned} \quad (3.21)$$

where $J_n(\alpha)$ is the ordinary Bessel function and $U_n(\xi)$ is the Chebyshev polynomial of the second kind.

On the basis of the viscous analysis of a zero thickness surface, (Ref. 9) the value of the circulation Γ must be such that the inviscid flow at the trailing edge is smooth (the Kutta condition). In that case the expression in square brackets in (3.19) must vanish as $x \rightarrow 1^-$. The resulting expression for Γ is,

$$\Gamma_K = - \frac{2 \int_{-1}^1 [(1+s)(1-s)]^{\frac{1}{2}} \mathcal{W}(s) ds}{ik e^{ik} [K_1(ik) + K_0(ik)]} \quad (3.22)$$

where K_0, K_1 are modified Bessel functions of the second kind. The subscript on Γ is used to indicate the Kutta condition.

On the other hand, if the surface has a finite thickness, the value of Γ is diminished in a way that must be calculated with the viscous theory. An extreme case is that when no vorticity is shed into the wake or $\Gamma = 0$. One may refer to this example as a case of no Kutta condition. It is physically the more correct result for high frequency finite thickness surface interactions where there is insufficient time for viscosity to establish a finite circulation. Recall (Figure 2), however, that even at low frequency there are significant departures of the total lift from the result produced by the Kutta condition.

The expression for the total lift on the surface is

$$L = \Gamma \left[ik e^{ik} K_1(ik) \right] - 2ik \int_{-1}^1 (1 - s^2)^{\frac{1}{2}} \mathcal{W}(s) ds \quad (3.23)$$

For $k \rightarrow 0$ the lift is proportional to Γ . For high frequency and no Kutta condition the lift is given by

$$L_{NK} = -2ik \int_{-1}^1 (1 - s^2)^{\frac{1}{2}} \mathcal{W}(s) ds \quad (3.24)$$

On the other hand if the full Kutta condition is applied at high reduced frequency, the asymptotic form of the lift is given by the expression

$$L_K \sim - \int_{-1}^1 [(1+s)/(1-s)]^{\frac{1}{2}} \mathcal{W}(s) ds - 2ik \int_{-1}^1 (1-s^2)^{\frac{1}{2}} \mathcal{W}(s) ds \quad (3.25)$$

The spectral form of the total lift is of interest because it is proportional to the far field acoustic spectrum in the compact limit. Consider the compact case where the reduced frequency is large and the Helmholtz number κ in (3.21) is small or

$$\kappa \ll 1 \ll k \quad (3.26)$$

The Mach number must, of course, be small for the validity of these relations. Then only the leading term in the series expression in (3.21) is important and $S(\alpha)$ is given by the asymptotic expression

$$S(\alpha) = \Gamma i k (\pi/2k)^{\frac{1}{2}} \exp(-i\pi/4) - 2ik \int_{-1}^1 (1 - s^2)^{\frac{1}{2}} \mathcal{W}(s) ds \quad (3.27)$$

With the full Kutta condition and no Kutta condition, the asymptotic spectrum is proportional to L_K and L_{NK} , respectively.

For convective disturbances (e.g., a frozen turbulence pattern that interacts with the surface) the spectrum of the surface velocity is of the form

$$\mathcal{W}(s) = \mathcal{W}_0 e^{-iks} \quad (3.28)$$

where \mathcal{W}_0 is some function of the frequency. In this case the integral in (3.27) can be evaluated exactly and the acoustic spectra and lift are given by the following expressions:

$$S_K = L_K = -2 \mathcal{W}_0 (2\pi/k)^{\frac{1}{2}} e^{i(k - \pi/4)} \quad (3.29)$$

$$S_{NK} = L_{NK} \cong -4 \mathcal{W}_0 (2\pi/k)^{\frac{1}{2}} \cdot i \sin(k - \pi/4) \quad (3.30)$$

and for the spectral density functions,

$$|S_K|^2 = |L_K|^2 = 8\pi |\mathcal{W}_0|^2 \cdot 1/k \quad (3.31)$$

$$|S_{NK}|^2 = |L_{NK}|^2 = 8\pi |\mathcal{W}_0|^2 \cdot \frac{4}{k} \sin^2(k - \pi/4) \quad (3.32)$$

The interesting feature of these results is that with a Kutta condition the spectrum decays monotonically with k while the no Kutta spectrum has an oscillatory decay with local maxima and zeroes for particular values of k as

shown in Figure 3. These results suggest that a simple experiment could be derived to measure the acoustic spectrum of a controlled convective or acoustic disturbance and determine by what degree the Kutta or no Kutta behavior is prevalent at high frequency.

B. Vortex Fly By a Sharp Edge

A specific example of the foregoing result is that of a single weak vortex convecting past a fixed surface. In that case the surface upwash is

$$\mathcal{W}(x,t) = \frac{\Gamma_0}{2\pi v_\infty} \frac{x - v_\infty t}{(x - v_\infty t)^2 + h^2} \quad (3.33)$$

where Γ_0 is the strength of the vortex and h is the vertical distance of the vortex from the plate. The spectrum of \mathcal{W} is of the form (3.28) with

$$\mathcal{W}_0 = \frac{\Gamma_0}{2v_\infty^2} e^{-k\tau} \quad (3.34)$$

where

$$\tau = 2h/c \quad (3.35)$$

and k is assumed to be positive. For convecting boundary layer turbulence, the parameter h is of the order of the boundary layer thickness, the spectrum is peaked and has a sharp cutoff near the reduced frequency, $k = 1/\tau$, that is much greater than unity. Substitute (3.34) into (3.31) and (3.32) and estimate the acoustic intensity with the relation

$$\mathcal{I} = (2v_\infty/c)^2 \frac{\rho}{a} \int_0^\infty |h'_a|^2 dk \quad (3.36)$$

The final result is

$$\mathcal{I} = \frac{\rho v_\infty^6}{2\pi a^3} \cdot \frac{l^2}{\gamma^2} \cdot \sin^2 \theta \left(\frac{\Gamma_0}{c v_\infty} \right)^2 \int_0^\infty k e^{-2k\tau} F^* dk \quad (3.37)$$

where

$$F^* = 1 \quad \text{Kutta Condition}$$

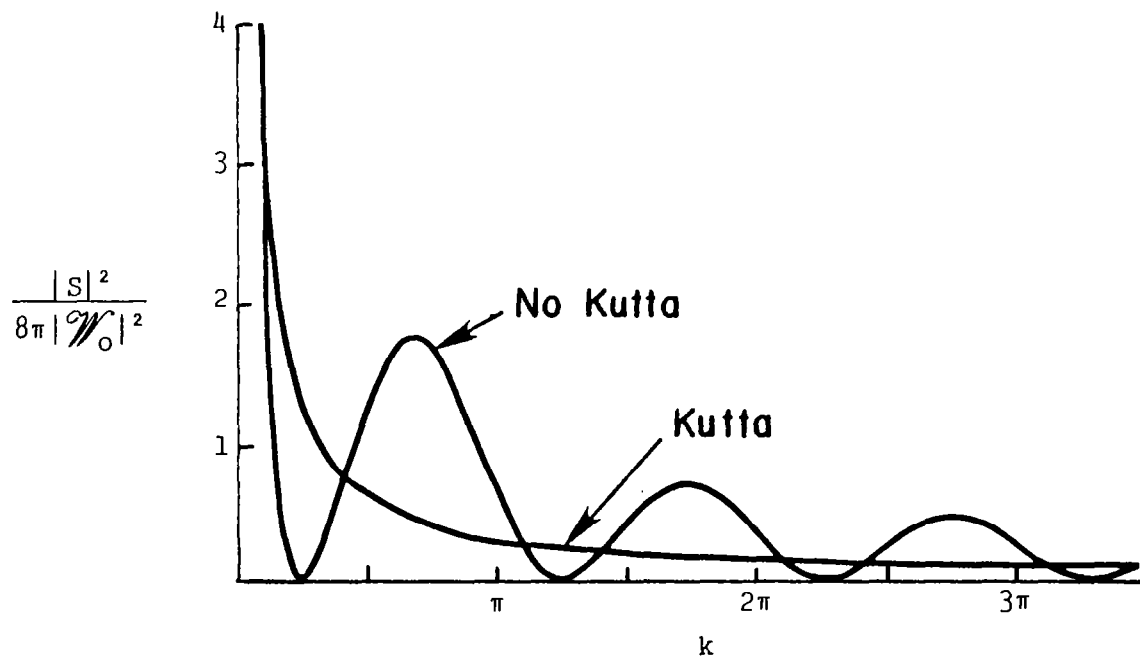


Figure 3 - Spectrum of the Far Field Interactive Noise and Surface Load for an Airfoil Encountering a Sinusoidal Gust; $k = \omega c / 2v_\infty$.

$$= 4\sin^2(k - \pi/4) \quad \text{No Kutta Condition} \quad (3.38)$$

The integration over k is easily performed with and without the Kutta condition. The ratio of the acoustic intensity with and without the Kutta condition is

$$\frac{\mathcal{I}_K}{\mathcal{I}_{NK}} = \frac{1}{(2 - 4\tau^3)/(1 + \tau^2)^2}$$

$$\approx 1/2 \quad \text{for } \tau \ll 1 \quad (3.39)$$

Thus the intensity is approximately 3dB less when the Kutta condition is applied. With a full Kutta condition convective vorticity is produced in the wake with an energy level that is the same order of magnitude as the radiated sound. In the supplement to this report a full viscous problem with no flow is considered and a similar result is obtained. The vortical energy in that case, however, is confined to the vicinity of the edges and is not convected into the far wake as above. An even more drastic result was obtained in the recent trailing edge noise study of Howe (Ref. 13). For a semi-infinite plate the edge noise due to convected turbulence is typically 10dB greater when no Kutta condition is applied. It should be pointed out however that the semi-infinite plate presents a much more singular problem because of the possibility of very weak upstream decay of the surface loading.

IV. SUPPLEMENTARY MATERIAL

A. Discussion of Trailing Edge Noise Models

During the earlier part of this program an inviscid dynamic model of discrete vortex fly by of a blunt (parabolic and rectangular) trailing edge was developed and calculations of the surface pressure and spectra near the trailing edge were made. The objective was to compare with the experimental results of Brooks and Hodgson (Ref. 6). The inviscid model accounts for two effects that are not included in the preceding analysis. First, the geometric shape of the edge (and the associated curvature of the mean flow) and second, the non-linear reaction of the convecting vortex to the surface boundary conditions. On the other hand, the important effect of vortex production and shedding due to viscosity is omitted.

The discrete vortex model was used in earlier work (Refs. 1,2) to illustrate the basic coupling between the vorticity and acoustic modes. The model has been used by Hardin (Ref. 14) to model the hydrodynamic origin of cavity noise. More recently Obermeier (Ref. 15) has used the model to calculate the noise emanated due to vortex fly by of finite cylindrical bodies. Again, no vortex shedding is permitted in Obermeier's study and noise is calculated with the theory of Möhring (Ref. 16) that in practice differs very little from the theory of Powell (Ref. 17) as used by Hardin (Ref. 14).

While some of the basic velocity scaling laws and frequency parameters evolve from the inviscid discrete vortex model, the fundamental question of the trailing edge noise problem is side stepped completely. That is, how does viscosity control the vortex shedding and load alleviation near the edge? Howe, (Ref. 13) in a recent review of trailing edge noise theory has called attention to the Kutta condition as the fundamental uncertainty in the edge noise problem (See Section IIIB). If one adopts a potential theory point of view with regard to the trailing edge fluid mechanics, and, furthermore, if the edge is mathematically sharp, then the difficulty can indeed be reduced to an argument about the potential flow edge singularity or the Kutta condition. However, it is this authors point of view that "potential theory" as a fluid mechanical model is at its very worst near the trailing edge. Furthermore, with the slightest amount of edge blunting the edge singularity is eliminated and the Kutta condition becomes meaningless as a local vortex shedding criterion. More important is the fact that singularity arguments avoid rather than illuminate the basic physics of the problem. The main thesis of the present work is that viscosity is the missing piece of the physics and must be accounted for in the fluid mechanical calculation near the trailing edge.

It was the realization of the importance of direct viscous effects in this study and in Ref. 8 that led to a redirection of the present effort from potential flow non-linear modeling to viscous flow linear modeling. Ideally, it would be desirable to calculate the complete nonlinear viscous fluid mechanics but that, of course, is an extremely difficult problem at the present time. The present state of development of the linear viscous model is the following: The theory for the zero thickness two-dimensional surface is complete and some calculations have been made for low reduced frequency. The results are more directly applicable to unsteady airfoil work and are presented in Ref. 8. The evaluation of the viscous kernel function and the load calculations for high frequency is not trivial but considerable progress has been made.

Based on the flat plate results for low frequency it was found, however, that direct viscous effects at the trailing edge are so strong that the calculations converge to the inviscid results with Kutta condition for Reynolds number (based on the chord) greater than 500.

The foregoing result means that when direct viscous effects are included in the linearized problem, there must not be any geometric length introduced that forces an infinitesimal or zero Reynolds number as for example at a mathematically sharp trailing edge. The correct formulation of the linearized viscous problem must include the finite trailing edge geometry, as discussed in Ref. 8. With a finite edge geometry the evaluation of the high frequency viscous function is even more difficult. To date only low frequency results have been obtained for oscillating rigid airfoils. The load calculations agree more closely with experimental results (Ref. 10) (both in magnitude and phase) than the flat plate calculations with Kutta condition. Based on the low frequency results it is concluded that even greater departures from the flat plate theory will result as the frequency is increased. The fundamental linearized problem, then, is to solve for the viscous load distribution on a surface with finite geometry for frequencies well into the acoustic regime. The basic theory is available and it remains a computational problem to carry out this program.

B. Summary of the Supplemental Report

In Ref. 18, explicit viscous calculations have been made for an oscillating airfoil problem. The results are extremely important both for the conceptual understanding of noise-surface interaction problems and for the quantitative calculation of interactive noise. The problem was motivated by the experiment of Brooks (Ref. 5). An airfoil section was vibrated at Helmholtz numbers of 1.3 and 2.04 in an anechoic chamber. The surface pressure and motion and the radiated pure tone sound were measured simultaneously. The surface measurements were also used in the "inviscid" Kirchhoff integral relation to calculate the radiated sound. The calculation overestimated the sound by 2 to 5 dB depending on direction. Also the measured chordwise surface pressure distribution has a flat behavior near the edges while aerodynamic theory predicts a square root decay of the edge loading. A more detailed description of the experiment may be found in (Ref. 5).

The theoretical work in the supplementary report has shown that the measured surface load and sound field are mutually consistent if the effect of viscous dissipation at

the edges is taken into account. The inviscid Kirchhoff integral has acoustic energy conservation built in so that all of the work done by the vibrating airfoil is propagated to the far field in the form of sound. When the same surface load is used in a viscous incompressible calculation the surface work appears in the near field (close to the edges) as kinetic energy associated with the vorticity formation due to viscosity. For a Helmholtz number of order unity and a near quadrupole distribution of the surface load it is shown that the viscous energy dissipation is nearly equal to that propagated as sound.

For Helmholtz numbers less than unity (into the compact regime) it is expected that even more energy will be transferred into the vorticity mode. Inviscid acoustic theory in the form of Kirchhoff integrals will in general overpredict the noise in the compact regime. In each application an estimate of the dissipation should be made with an incompressible viscous analysis like that in the supplementary report. A correction to the inviscid Kirchhoff calculation can then be made. For very small Helmholtz numbers it would appear that there is no alternative to a complete compressible viscous calculation. Furthermore, when there is a finite convection velocity the surface work transferred into vorticity is greatly enhanced and it is even more important to understand and calculate the effect of viscosity.

V. CONCLUSIONS

The most important conclusion of this study is that viscosity is an important missing physical element in aeroacoustic theories where interactions with a surface with edges is involved. Even in the absence of a mean flow there can be an appreciable viscous effect near the edge with an associated loss of energy to the acoustic mode. With a finite convection velocity, the magnitude of the circulation around the surface is determined uniquely by viscosity. The use of the Kutta condition will in general overestimate the circulation which means that more vorticity is shed into the wake and less noise is generated. There is no *a priori* reason why a potential flow singularity cannot occur at the trailing edge. It is much more important to estimate the magnitude of the load distribution correctly over the entire surface and this can be accomplished with the viscous theory. It is important that finite geometry and boundary layer displacement thickness effects be accounted for in the viscous calculation. The acoustic theory developed in this report and the associated report on viscous thin airfoil theory (Ref. 8) is the correct approach for carrying out the surface load calculation. The difficult practical step is the unsteady aerodynamic load calculation for high reduced frequency - a problem that has perplexed the unsteady aerodynamicist for many years.

Considerable progress has been made during the course of this program but much work remains to be done before routine calculations can be made.

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16. Abstract A viscous linear surface noise interaction problem is formulated that includes noise production by an oscillating surface, turbulent or vortical interaction with a surface, and scattering of sound by a surface. The importance of viscosity in establishing uniqueness of solution and partitioning of energy into acoustic and vortical modes is discussed. The results of inviscid two-dimensional airfoil theory are used to discuss the interactive noise problem in the limit of high reduced frequency and small Helmholtz number. It is shown that in the case of vortex interaction with a surface, the noise produced with the full Kutta condition is 3 dB less than the no Kutta condition result. Also, the spectrum with Kutta condition decays monotonically with frequency while the corresponding result without the Kutta condition decays in an oscillatory manner. The results of a supplementary study of an airfoil oscillating in a medium at rest are discussed. It is concluded that viscosity can be a controlling factor in analyses and experiments of surface noise interaction phenomena. It is further concluded that the effect of edge bluntness as well as viscosity must be included in the problem formulation to correctly calculate the interactive noise.					
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